Royal University of Bhutan

Paro College of Education

Autumn Semester Examination, 2014

B. Ed (S) II - Linear Algebra (MAT 205)

Full Mark : 100 **Time : 3 Hours**

Instruction

Do not write for the first TEN minutes. This time is to be spent reading the questions.The above mentioned time is for writing the answer.

The question paper consists of TWO sections. Answer ALL questions from section A and any FIVE Questions from section B.

You are allowed to use a scientific calculator of **fx-82 or fx-100** *beside other writing materials. Graph sheets will be provided if it is required.*

Section A (40 marks)

Attempt all sub-questions in this question. Each sub questions carry 4 marks.

Question 1

a. State Triangle and Parallelogram Law of Vectors Addition.

b. Find the direction cosine and unit vector in the direction of the sum of the vectors, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$.

c. If $\vec{a}$ and $\vec{b}$ are any two vectors and m be a scalar, then show that

$$m(\vec{a} \times \vec{b}) = (m\vec{a}) \times \vec{b} = \vec{a} \times (m\vec{b}).$$

d. Define the following terms of Linear Programming:

(i) Feasible region; (ii) constraints;

(iii) Optimal Feasible solution; (iv) Convex region.

e. A photographer charges a base fee of Nu. 250 and then charges Nu. 50 for each photograph ordered. Sonam can afford to spend no more than Nu. 1060. What different quantities of photos can Sonam afford? Use number line to represent the different quantities of photos.

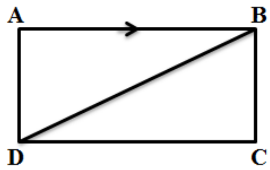
f. Define symmetric and skew symmetric matrices with appropriate examples.

g. Using row reduction method, find the inverse of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$.

h. Solve for X

$$\begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} X + \begin{pmatrix} -5 & 0 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -9 \\ 7 & 1 \end{pmatrix}.$$

i. The digraph below shows direct flights among four airports.



(i) Create an adjacency matrix.

(ii) Use the matrix to calculate the one-stopover and two stop-over trips from A to C .

j. Show that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0.$

Section B (60 marks)

There are SIX questions in this section. Attempt only FIVE questions. Each question carries 12 marks. You must show all working steps for each question.

Question 2

a. Show that the points $A(2\hat{\imath} - \hat{\jmath} + \hat{k}),\ B(\hat{\imath} - 3\hat{\jmath} - 5\hat{k}),\ C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$ are the vertices of a right angled triangle. **(4)**

b. Evaluate whether the following points are collinear or non-collinear points $A(3, -5, 1)$, $B(-1, 0, 8)$ and $C(7, -10, -6)$. **(4)**

c. Using scalar product, prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B$. **(4)**

Question 3

a. Prove that the line joining the mid-points of two sides of a triangle is parallel to the third side and half of it. **(3)**

b. Prove that the vector area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}(\vec{a} \times \vec{b})$. Using the proved result, find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = -3\hat{\imath} - 2\hat{\jmath} + \hat{k}$. **(5)**

c. Draw the graph and shade the feasible region of the following system of linear inequalities $x + y \leq 5$, $4x + y \geq 4$, $x + 5y \geq 5$, $x \leq 4$ and $y \leq 3$. **(4)**

Question 4

a. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

From/To	**Transportation cost per quintal (in Rs))**		
	D	E	F
A	6	3	2.5
B	4	2	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost? **(6)**

b. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week? **(Solve by Iso-cost method)** **(6)**

Question 5

a. Using Gaussian elimination method, test for consistency and give the solution (if it exists) for the system of equations $x+y+z = 7$; $x+2y+3z = 16$; $x+3y+4z = 22$.

(6)

b. Find the eigen vector and corresponding eigen vectors for the matrix **(6)**

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

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Question 6

a. If A and P are square matrices of the same order and P is non-singular, then show that A and $(P^{-1}AP)$ have the same character roots. **(4)**

b. State and prove the reversal law of transpose. **(4)**

c. Without expanding the determinant, show that $(a+b+c)$ is a factor of the following determinant: **(4)**

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Question 7

a. Solve $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-9 & 3x-64 \end{vmatrix} = 0.$ **(6)**

b. Solve the following system of equations using Cramer's rule: **(6)**

$$5x - 7y + z = 11,\ 6x - 8y - z = 15,\ 3x + 2y - 6z = 7$$