Royal University of Bhutan

Paro College of Education

Spring Semester Examination, 2014

B. Ed (S) II - Algebra and Trigonometry (MAT 203)

Full Mark : 100 **Time : 3 Hours**

Instruction :

Do not write for the first ten minutes. This time is to be spent in reading the questions. The question paper consists of TWO sections. The above mentioned time is for writing the answer.

You are allowed to carry a scientific calculator of **fx-82 or fx-100** *beside other writing materials. Graph sheets will be provided if it is required.*

Section A (10 X 3 = 30 marks)

Attempt all questions in this section.

Question 1 :

i. State the "Principle of Extension" of sets and give an example in set builder form.

ii. If A, B and C are finite and joint sets, then show that the number elements only in A is given by

$$|A - (B \cup C)| = |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$

iii. Define the following term;

(a) Boolean Algebra, (b) Additive set function.

iv. State the **fundamental principle** of Addition and Multiplication law of counting.

v. In how many ways can 6 gentlemen and 3 ladies be seated around a table so that every gentlemen may have a lady by his side?

vi. There are 6 red balls, 4 green balls and 7 blue balls. In how many ways can a selection of balls be made if

(a) balls of same kind are different,

(b) balls of the same kind are identical?

vii. Prove using mathematical induction that for all $n \geq 1$

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right).....\left(1 - \frac{1}{(n+1)^2}\right) = \frac{n+2}{2n+2}$$

viii. Find the middle term(s) in the expansion of $\left(2x^2 + \frac{1}{x}\right)^9$.

ix. Prove that $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = 2\cos\theta$.

x. Show that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \frac{x + y + z - xyz}{1 - xy - yz - zx}$.

Section B (5 X 14 = 70 marks)

There are SIX questions in this section. Attempt only FIVE questions. Each question carries 14 marks. You must show all working steps for each question.

Question 2 :

i. If A and B are finite joint sets and $A \cup B$ is finite, then show that **(5 marks)**

$$|A \cup B| = |A| + |B| - |A \cap B|$$

ii. In a college, out of 96 students, 48 offered Mathematics, 45 offered IT, 52 offered Physics, 15 offered Mathematics and IT, 18 offered Mathematics and Physics, 21 offered IT and physics. Each student offered at least one subject and x students offered all the three subjects. Illustrate the sets on a clearly labelled Venn diagram. In the your diagram, show x and the number of members in each region. Use Venn diagram to find the number of students who

(a) offered all subjects,
(b) offered only mathematics
(c) offered only one subject.
(d) offered only two subjects. **(6 marks)**

iii. Draw a Venn diagram to show the relationship between the following sets
A = {2 D shapes} B= {quadrilaterals} C = {Parallelogram}
D = {Rectangles} E = {Rhombuses} F = {Triangles}
Show in your diagram the region that represents the set of square. **(3 marks)**

Question 3 :

i. How many words can be made with letters of the word INTERMEDIATE if

(a) the words neither begin with I nor ends with E
(b) the vowels and consonants alternate in the words
(c) no vowels is between two consonants. **(6 marks)**

ii. In how many ways can 20 students be divided into four equal groups? In how many ways can these be sent to four different schools? **(2 marks)**

iii. How many triangles and rectangles may be formed by joining any of the nine points when

(a) all are non-collinear

(b) five of them are collinear. **(6 marks)**

Question 4:

i. Out of 3 books on Math, 4 books on IT and 5 books on English, how many collections can be made, if each collection consists of

(a) exactly one book on each subject,

(b) at least one book on each subject? **(4 marks)**

ii. Using principle of mathematical induction, prove that $\left(\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}\right)$ is a natural number for all natural numbers n. **(5 marks)**

iii. Prove by induction that $\sum_{r=0}^{n} r \cdot {}^{n}C_r = n \cdot 2^{n-1}, \ \forall\ n \in \mathbb{N}$. **(5 marks)**

Question 5 :

i. State and proof Binomial theorem for positive integral index. **(5 marks)**

ii. Given that $(1+x)^n = C_0+C_1x+C_2x^2+......+C_nx^n$ and $C_0+C_1+C_2+....+C_n = 512$. Find the value of n. Using the determined value of n, and find the term independent of x in the expansion $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^n$. **(4 marks)**

iii. Prove that $C_1x+2C_2x^2+3C_3x^3+.....+nC_nx^n = nx(1+x)^{n-1}$. Using this resultant, find the value of $C_0+2C_1x+3C_2x^2+4C_3x^3+.....+(n+1)C_nx^n$. **(5 marks)**

Question 6 :

i. Identify the following series with binomial expansion and hence find the sum of the series $1+\frac{2}{5}+\frac{1\cdot 3}{2!}\left(\frac{2}{5}\right)^2++\frac{1\cdot 3\cdot 5}{3!}\left(\frac{2}{5}\right)^3+.....$ **(4 marks)**

ii. With explanation , find the t-ratio of $(90^0-\theta)$ for sin, cos and tan. **(4 marks)**

iii. Prove that

(a) $\dfrac{\cos 3A+2\cos 5A+\cos 7A}{\cos A+2\cos 3A+\cos 5A} = \cos 2A-\sin 2A\tan 3A$

(b) $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}x^2$ **(6 marks)**

Question 7 :

i. Prove that $\cos^2 A + \cos^2(A+120^0) + \cos^2(A-120^0) = \dfrac{3}{2}$ **(4 marks)**

ii. Prove that $\sin 3A = 4\sin A \sin\left(\dfrac{\pi}{3} - A\right)\sin\left(\dfrac{\pi}{3} + A\right)$. Hence show that $\sin\left(\dfrac{\pi}{9}\right)\sin\left(\dfrac{2\pi}{9}\right)\sin\left(\dfrac{3\pi}{9}\right)\sin\left(\dfrac{4\pi}{9}\right) = \dfrac{3}{16}$ **(4 marks)**

iii. Solve the equation $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\dfrac{8}{31}$. **(3 marks)**

iv. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, prove that $a+b+c = abc$. **(3 marks)**