Spring Semester Examination 2019
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 205 (Linear Algebra) **Programme:** B.Ed(S) **Level** : II

Writing Time: Three Hours **Full Marks:** 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions.Write the answers to all the questions in the answer sheets provided by the college.Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator ***fx-82 or fx-100*** beside other writing materials. You will be provided with graph sheets.

Section A
One Question - 40 marks

Question 1

Instructions: Attempt all sub-questions in this question. Each sub questions carry 4 marks.

a. State Triangle and Parallelogram Law of Vectors Addition.

b. Find the unit vector in the direction of the sum of the vectors, $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$.

c. Prove that the straight line joining the mid-points of two non-parallel sides of a trapezium is parallel to the parallel sides and half of their sum.

d. Define the following terms of Linear Programming:

(i) Feasible region; (ii) constraints;
(iii) Optimal Feasible solution; (iv) Convex region.

e. Draw graph of the solution set of the following system of linear inequations:

$$y \geq -x + 4, 2y \geq x - 1, 3y \leq -x + 2, x \geq 1, y \leq 6$$

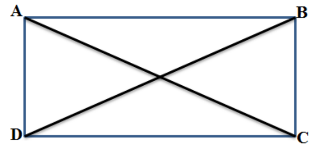
f. Define symmetric and skew symmetric matrices with appropriate examples.

g. Using row reduction method, find the inverse of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$.

h. Solve for X

$$\begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} X + \begin{pmatrix} -5 & 0 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -9 \\ 7 & 1 \end{pmatrix}.$$

i. The digraph below shows direct flights among four airports.



(i) Create an adjacency matrix.

(ii) Use the matrix to calculate the two stop-over trips from A to C .

j. Show that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0.$

Section B
Five Questions - 60 marks

Instructions: There are SIX questions in this section. Attempt only FIVE questions. Each question carries 12 marks. The intended mark for each sub-question is given in the brackets. You must show all working steps for each question.

Question 2

a. Show that the points $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$, $C(3\hat{i} - 4\hat{j} - 4\hat{k})$ are the vertices of a right angled triangle. **(6)**

b. Using scalar product, prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B$. **(6)**

Question 3

a. Show that quadrilateral is a parallelogram if its diagonals bisect each other. **(5)**

b. Prove that the vector area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}(\vec{a} \times \vec{b})$. Using the proved result, find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$. **(7)**

Question 4

a. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week? **(7)**

b. Solve the following linear programming problem graphically using the Iso-cost method. **(5)**
Minimize $Z = 3x + 5y$ subject to the constraints

$$-2x + y \leq 4,\ x + y \geq 3,\ x - 2y \leq 2,\ x \geq 0,\ y \geq 0$$

Question 5

a. Solve the following system of equations, using the matrix method: **(6)**

$$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11$$

b. Find the Eigen values and corresponding Eigen vectors for any one of the Eigen values for the matrix **(6)**

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

Question 6

a. Determine the consistency of by the matrix method and find solution if it is consistent. **(6)**

$$2x - y + 3z = 5,\ 3x + 2y - z = 7,\ 4x + 5y - 5z = 9$$

b. Without expanding the determinant, show that $(a+b+c)$ is a factor of the following determinant: **(6)**

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Question 7

a. Solve $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-9 & 3x-64 \end{vmatrix} = 0.$ **(6)**

b. Solve the following system of equations using Cramer's rule: **(6)**

$$5x - 7y + z = 11,\ 6x - 8y - z = 15,\ 3x + 2y - 6z = 7$$