Autumn Semester Examination 2015
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 203 (Algebra and Trigonometry) **Programme:** B.Ed(S) **Level** : II
Writing Time: Three Hours **Full Marks:** 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions.
Write the answers to all the questions in the answer sheets provided by the college.Read the directions to each section and to each question carefully before answering the questions.
You are allowed to carry a scientific calculator of ***fx-82 or fx-100*** beside other writing materials.

Section A
TEN Questions - 40 Marks

Instructions : Attempt all the questions in this section.

Question 1

a. Define Disjoint and Complimentary sets with the help of Venn diagrams.

b. If A and B are two sets,then prove that $A \times B = B \times A$ if and only if $A = B$, or $A = \phi$, or $B = \phi$.

c. Show that, if $\sigma = \{\phi, \xi, S, S'\}$ where S is any non-empty subset of ξ, the universal set, the system $[\sigma, \cap, \cup]$ is a Boolean algebra.

d. Explain OR and AND switching circuits with proper circuit diagrams and truth tables.

e. A round table conference is to be held among delegates of 20 countries. In how many ways can they be seated if two particular delegates are

   i. always together?
   ii. never together?

f. In how many ways can playing cards can be

   i. placed in 4 heaps equally?
   ii. dealt out to four players equally?

g. Using the principle of mathematical induction, prove that $10^n + 3 \cdot 4^{n+2} + 5$ is divisible by 9, for all $n \in \mathbb{N}$.

h. If C_0, C_1, $C_2, \ldots, C_n$ be the coefficients in the expansion of $(1+x)^n$, prove that

$$C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{2n!}{n!\,n!}.$$

i. Prove that $\cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0 = \dfrac{1}{16}$.

j. Solve $\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{65}$.

Section B

SIX Questions - 60 Marks

Instructions : There are SIX questions in this section. Attempt any FIVE. Each question carries 12 marks. You must show all working steps for each question.

Question 2

a. State and prove De Morgan's laws of sets using both Venn diagrams and analytic methods. **(4)**

b. In a college, 65% of the students play Football, 54% play Volleyball, 45% play Basketball; 38% play Football and Volleyball, 32% play Volleyball and Basketball, 28% play Football and Basketball; 12% do not play any of the three games. If there are 10,000 students in the college, find the number of students who

i. play all three games;
ii. play at least two games;
iii. play only Football;
iv. play only two games. **(5)**

c. Draw a Venn diagram for each of the following relations :

i. Quadrilaterals, Parallelograms, Rhombi, Rectangles, Squares.
ii. All triangles, Isosceles triangle, Equilateral triangle, Right triangle, Acute-angled triangle
iii. PCE, PgDE, B.Ed, 2MIT, 2PCE. **(3)**

Question 3

a. In how many different ways can the letter of the word SALOON be arranged
i. if the two O's must come together?
ii. if the consonants and vowels must occupy alternate places? **(4)**

b. Out of 3 books on Economics, 4 books on Political Science and 5 books on Mathematics, how many collections can be made, if each collection consists of:

   i. exactly one book on each subject?

   ii. at least one book on each subject? **(4)**

c. A boat is to be manned by 9 crew members with 4 on the stroke side, 4 on the row side and one to steer. There are 11 crew of which 2 can stroke only, 1 can row only while 3 can steer only. In how many ways the crew can be arranged for the boat? **(4)**

Question 4

a. A committee consists of three members. The committee passes a resolution when at least two of the members vote 'yes'. Write a Boolean function symbolising the four possible combinations, with a closure table. By simplifying this function, design a circuit in which a lamp lights when a resolution is passed. **(6)**

b. If $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$, $n \in \mathbb{N}$ then using the principle of mathematical induction prove that $S_1+S_2+S_3+\ldots+S_n = (n+1)S_n - n$. **(6)**

Question 5

a. Find the middle term, term independent of x, and coefficient of $\frac{1}{x^6}$ in the expansion $\left(\sqrt{\frac{x}{3}} - \frac{\sqrt{3}}{2x}\right)^{12}$. **(4)**

b. Find the cube root of 1001 correct to five decimal places. **(4)**

c. Identify the series as binomial expansion and hence find its sum of the series $1 + \frac{1}{4} + \frac{1\cdot 3}{4\cdot 8} + \frac{1\cdot 3\cdot 5}{4\cdot 8\cdot 12} + \ldots$ **(4)**

Question 6

a. From the top of a building, 60 metres high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 40^0 and 60^0 respectively. Find

   i. the horizontal distance between the building and lamp post;

   ii. the height of the lamp post. **(4)**

b. Prove that

   i. $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$ **(4)**

   ii. $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$. **(4)**

Question 7

a. Prove that $\cos\theta + \cos(120^0 + \theta) + \cos(\theta - 120^0) = 0$. Deduce that $\cos^3\theta + \cos^3(120^0 + \theta) + \cos^3(\theta - 120^0) = \dfrac{3}{4}\cos 3\theta$ **(5)**

b. Solve the equation $\sin^{-1} 6x + \sin^{-1}(6\sqrt{3}x) = -\dfrac{\pi}{2}$ **(3)**

c. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$. **(4)**