Autumn Semester Examination 2017
Paro College of Education
Royal University of Bhutan
Paro

**Module** : MAT 409 (Integral Calculus)    **Programme:** B.Ed(S)    **Level**    : IV

**Writing Time:** Three Hours                                        **Full Marks:** 100

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**Instructions :** Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator *fx-82 or fx-100* beside other writing materials.

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**Instructions :** This paper contains FIVE questions. Answer any FOUR questions. All questions carry 25 marks each. Marks for each question or sub question are given in the brackets.

**Question 1**

a. Evaluate $\displaystyle\int \frac{dx}{\sin(x-a)\cos(x-b)}$.    [6]

b. Prove that $\displaystyle\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$.    [6]

c. Find the area of a region bounded by $y = |x+1| + 1$, $x = 3$ and $y = 0$.    [6]

d. Prove that $\displaystyle\int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C$. Hence using the relation, evaluate $\displaystyle\int \sqrt{\frac{1+x}{x}}\, dx$.    [7]

**Question 2**

a. Find the area of a region bounded by the curve $y = x^2$ and the line $y = |x|$.    [6]

b. Evaluate $\displaystyle\int_1^3 (x^2 + 5x)\, dx$ as limits of sums.    [6]

c. Evaluate $\displaystyle\int (x+1)e^x \left[\log(xe^x)\right]^2 \, dx$.    [7]

d. Evaluate $\displaystyle\int \frac{e^x}{e^{4x} + 3e^{2x} - 4}\, dx$    [6]

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**Question 3**

a. Evaluate $\int \dfrac{3\cot x + cosec\, x}{1 + 2\cot x + 3\, cosec\, x}\,dx$. [6]

b. Find the area enclosed by the parabolas $y^2 = x$ and $y^2 = 4 - 3x$. [6]

c. Obtain a reduction formula for $\int \cos^m x \sin nx\,dx$ and evaluate $\int \cos^5 x \sin 3x\,dx$. [7]

d. Evaluate $\displaystyle\int_{e^{-2}}^{e^4} \log_e x \dfrac{\left[\log_e x\right]}{x}\,dx$ [6]


**Question 4**

a. Prove that $\displaystyle\int_0^{\pi/4} \log(1 + \tan x)\,dx = \dfrac{\pi}{8}\log 2$. [6]

b. Calculate the volume of the solid of revolution generated when the area of a region bounded by a curve $y = 2 - x^2$ and a straight line $\sqrt{2}x + y = 2$ is rotated around the $x$-axis. [7]

c. Evaluate $\int \dfrac{\sin\theta}{\cos\theta(\cos^5\theta + 1)}\,d\theta$. [6]

d. Evaluate $\int \dfrac{\sqrt{1 + x^2}}{1 - x^2}\,dx$. [6]


**Question 5**

a. Evaluate $\int x \sin^{-1} x\,dx$. [6]

b. Find the volume of the solid formed by the revolution of a region enclosed by the line $y = x$ and curve $y = x^3$ about the $x$-axis. [7]

c. Evaluate $\displaystyle\int_a^b \cos x\,dx$ as the limit of a sum. [6]

d. Evaluate $\int \dfrac{x^2}{x^4 + 1}\,dx$ [6]