

Autumn Semester Examination 2017  
Paro College of Education  
Royal University of Bhutan  
Paro

**Module:** MAT 203 (Algebra and Trigonometry)    **Programme:** B. Ed(S)    **Level:** II  
**Writing Time:** Three Hours    **Full Marks:** 100

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**Instructions:** Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions.

You are allowed to carry a scientific calculator of *fx-82* or *fx-100* beside other writing materials.

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**SECTION A - (10 × 4 = 40 Marks)**

**Instructions:** Attempt all the sub-questions in this section. All sub-questions carry equal marks.

**Question 1**

- a. Define intersection and union of sets with the help of Venn diagrams.
- b. Draw AND and OR switching circuits diagrams with truth tables.
- c. Express  $(\cos \theta - \sin \theta)$  as a cosine of an angle.
- d. Find the value of  $\sin 2x$  and  $\cos 2x$ , if  $\sin x = \frac{3}{5}$  and  $\cos x = \frac{4}{5}$ , such that  $0 < x < \frac{\pi}{2}$ .
- e. Find the radian measures that correspond to the degree measures  $300^\circ$  and  $-18^\circ$ .
- f. A restaurant offers 10 choices of drinks, 5 choices of main meal and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all three courses. Assuming all choices are available, how many different possible meals does the restaurant offer?
- g. Prove that  $(x-2)^3 = x^3 - 6x^2 + 12x - 8$  using binomial theorem expansion.
- h. Define the subset and list all the subsets of  $Q$ , if  $Q = \{2, 3, 5, 7\}$ .
- i. What are the different operations used in Boolean algebra? Describe the relation of each operation with appropriate logic gates.
- j. How many different three-letter logo can be made from the letters in the word 'FAITH' if:
  - i. repetition is not allowed?
  - ii. repetition is allowed?

**SECTION B - (60 Marks)**

**Instructions:** There are SIX questions in this section. Attempt any FIVE. Each question carries 12 marks. You must show all working steps for each question.

**Question 2**

- a. How is permutation different from combination? Explain with appropriate examples (6)
- b. Find the Cartesian product of the following sets and identify which of the six properties does this product matches. (6)

$$R = \{blue, red, green\} \text{ and}$$

$$S = \{peach, apple, orange\}$$

**Question 3**

- a. Prove that  $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{7\pi}{8} = \frac{3}{2}$  (6)
- b. Solve for  $x$ , if  $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$ . (6)

**Question 4**

- a. Prove that  $\left( \frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta = \frac{1 - \cos^2 \theta \sin^2 \theta}{2 + \cos^2 \theta \sin^2 \theta}$  (6)
- b. Prove that  $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ . (6)

**Question 5**

- a. List the elements of the following sets and answer the questions accordingly. Find: (6)

$$\xi = \{\text{Natural numbers less than 25}\}$$

$$A = \{\text{Even numbers less than 20}\}$$

$$B = \{\text{Prime numbers less than 20}\}$$

$$C = \{\text{Odd numbers less than 20}\}$$

$$D = \{\text{Composite number less than 20}\}$$

- i.  $n(A \cup B)$                       ii.  $(B \cup A)'$                       iii.  $(B \cap D)'$

- b. Design a word problem on your own and show the procedure of solving it using the Venn diagram. (6)

**Question 6**

- a. Prove that  $\cos^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{16}{65} = \frac{\pi}{2}$  (6)
- b. Prove that  $\cos^2 (A - B) + \cos^2 B - 2 \cos (A - B) \cos A \cos B$  is independent of  $B$ . (6)

**Question 7**

- a. If  $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ ,  $n \in N$ , then using the principle of mathematics induction prove that  $S_1 + S_2 + S_3 + \dots = (n + 1)S_n - n$ . (6)
- b. Look at the diagram/circuit below carefully and prove the statement  $A + AB = A$ . (6)

**A + AB = A**

