Autumn Semester Examination 2017
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 306 (Analytic Geometry) **Programme:** B.Ed(S) **Level** : III

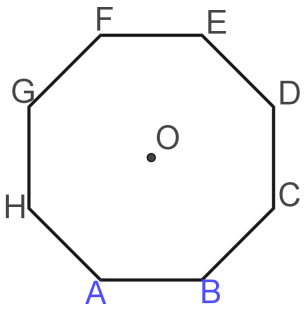
Writing Time: Three Hours **Full Marks:** 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator ***fx-82 or fx-100*** beside other writing materials. You will be provided with graph sheets.

Instructions:This paper contains FIVE questions. Answer any FOUR questions. All questions carry 25 marks each. Mark for each question or sub question is given in the bracket.

Question 1

a. The point O is the centroid of a regular octagon shown in the diagram below. Answer all sub-questions based on the given shape. [6]



Find the

i. angle of rotation,if the image of A is D.
ii. number of line symmetry.
iii. number of turn symmetry.
iv. number of reflectional symmetry.
v. number of plane of symmetry if it is the base of a pyramid.
vi. number of axis of rotation if it is the base of a prism.

b. State and prove the formula for angle(s) between two lines in their slopes form. Using the proved formula, find conditions of parallel and perpendicular of two lines. [7]

c. Explain different conics and their degenerated form with the help of appropriate diagrams of sections of a plane and a right circular cone. [6]

d. Find the equation of the circle which passes through the points $(5, 0)$ and $(1, 4)$, whose center lies on the line $x + y - 3 = 0$. Also find radius of the circle. [6]

Question 2

a. State and prove the theorem of reflection in the line $y = x$. Express the proved theorem into a matrix form. [6]

b. Find the equation of the line through the intersection of lines $x - y = 1$ and $2x - 3y + 1 = 0$, and parallel to the line $3x + 4y = 12$. [6]

c. Show that general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents parabola if $h^2 = ab$, ellipse if $h^2 < ab$ and hyperbola if $h^2 > ab$. How circle's equations are unique from the equation of conics? [7]

d. Find the equation of the circle which passes through the origin and cut off x-axis and y-axis at 3 and 4 respectively. What is radius and center of the circle? [6]

Question 3

a. If R is a reflection in y-axis and S is a reflection in x-axis,using matrices show that $RS = SR$. Which single transformation can replace the above composite transformations? [6]

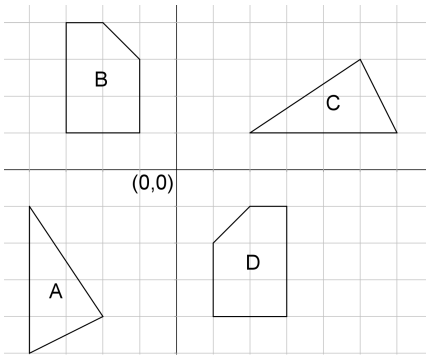
b. Define locus. Show that the equation of a locus $ax + by + c = 0$ is the equation of a straight line. [7]

c. Find the equation of the ellipse whose foci are at the points $S(2, 0)$ and $S'(-2, 0)$, and whose latus rectum is 6. [6]

d. Using a distance formula, show that the points $A(0, 1, 2)$, $B(2, -1, 3)$, and $C(1, -3, 1)$ are the vertices of the isosceles right angled triangle, and find its area. [6]

Question 4

a. Describe a composite transformation for each. Write each transformation in mapping notation and a single mapping for the composite transformation for each. [6]



i. mapping A onto C

ii. mapping B onto D

b. Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle α, given by the equation $\tan\alpha = \frac{5}{12}$ with the positive direction of the x-axis. [6]

c. Find the equation of the parabola whose axis is $3x - 4y + 5 = 0$, the vertex at $(1, 2)$ and the length of the latus rectum is 8. [6]

d. A, B, C are the points $(1, 4, 2)$, $(-2, 1, 2)$, $(2, -3, 4)$. Find the angles of the ΔABC.[7]

Question 5

a. Let $A(5, 5)$, $B(4, 2)$, $C(8, 3)$ be the vertices of a ΔABC and its image $\Delta A'B'C'$ has the vertices $A'(-3, 1)$, $B'(0, 0)$, $C'(-1, 4)$. Draw the shapes on the graph sheet provided to you and find the center and angle of rotation graphically. Verify your answer by using any one of the vertex and its image. [7]

b. Let $A(2, 0)$ and $B(4, 0)$ be two given points. A point P moves so that $PA^2 + PB^2 = 10$. Find the locus of P. What shape do we get from the locus? [6]

c. Find the centre, vertices, foci and eccentricity of the hyperbola
$$9x^2 - 16y^2 - 18x - 64y - 199 = 0$$ [6]

d. If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction cosines of two lines with angle θ between them, then prove that $\cos\theta = l_1 l_1 + m_1 m_2 + n_1 n_2$. [6]