Autumn Semester Examination 2017
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 307 (Differential Calculus) **Programme:** B.Ed(S) **Level** : III

Writing Time: Three Hours **Full Marks:** 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions. Write the answers to all the questions in the answer sheets provided by the college. Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator ***fx-82 or fx-100*** beside other writing materials.

Instructions : This paper contains FIVE questions. Answer any FOUR questions. All questions carry 25 marks each. Marks for each question or sub question are given in the brackets.

Question 1

a. Using Sandwich theorem, prove that $\lim_{x\to 0}\dfrac{\sin x}{x} = 1$, where x is in radian. Using the proved relation, solve $\lim_{x\to 0}\dfrac{2(1-\cos x^0)}{x^2}$. [7]

b. Discuss the continuity of the function $f(x)$ at the point $x = \dfrac{1}{2}$.

$$f(x) = \begin{cases} x, \;, & \text{if } 0 \le x < 1/2 \\ \dfrac{1}{2}\;, & \text{if } x = 1/2 \\ 1-x, & \text{if } 1/2 < x \le 1 \end{cases}, \qquad [6]$$

c. Differentiate $\sin x^2$ with respect to x using ab-initio method. [6]

d. Show that of all rectangles inscribed in a given circle, the square has the maximum area. [6]

Question 2

a. Find the value of a and b, if $f(x) = \begin{cases} \dfrac{\sin ax}{\tan bx}\;, & \text{if } x < 0 \\ \dfrac{3}{2}\;, & \text{if } x = 0 \\ \dfrac{\log(1+ax)}{e^{bx}-1}, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$. [6]

b. Evaluate $\lim_{x\to 0} f(x)$ (if it exists), $f(x) = \begin{cases} \dfrac{3x}{|x| + 2x^2}, & x \neq 0; \\ 0\;, & x = 0. \end{cases}$ [6]

c. Find the equations of the tangent and the normal to $16x^2 + 9y^2 = 144$ at (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$. [6]

d. If $y = \dfrac{x \sin^{-1} x}{\sqrt{1-x^2}} + \log \sqrt{1-x^2}$, then prove that $\dfrac{dy}{dx} = \dfrac{\sin^{-1} x}{(1-x^2)^{3/2}}$. [7]

Question 3

a. Evaluate $\lim\limits_{x \to 0} \dfrac{(x+y)\sec(x+y) - y \sec y}{a^x - b^x}$ [7]

b. Show that the **Greatest integer function** $\lfloor x \rfloor$ is continuous at all points except at integer points. [6]

c. If $y = (\sin x)^{\tan x} + (x)^{\sqrt{x}}$, find $\dfrac{dy}{dx}$. [6]

d. Sand is pouring out from a pipe at the rate of $12\, cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand-cone increasing when the height is $4\, cm$? [6]

Question 4

a. Evaluate $\lim\limits_{x \to 0} \dfrac{[\log(3+x) - \log 3]\,[\log(3+x) - \log(3-x)]}{8^x - 4^x - 2^x + 1}$ [6]

b. Discuss the continuity of the function $f(x) = \begin{cases} \dfrac{\sin x}{\tan x}, & \text{if } x < 0 \\ 2x+3, & \text{if } x \geq 0 \end{cases}$. [6]

c. Differentiate $x^{\sin^{-1} x}$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$. [6]

d. Show that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles. [7]

Question 5

a. Evaluate $\lim\limits_{x \to a} \dfrac{\sin x - \sin a}{x - a}$ [6]

b. Show that $f(x) = \begin{cases} \dfrac{x-4}{|x-4|} + 1, & \text{if } x < 4 \\ \quad 3 \quad , & \text{if } x = 4 \\ \dfrac{x-4}{|x-4|} - 1, & \text{if } x > 4 \end{cases}$ is discontinuous at $x = 4$.

Discuss the type of discontinuity and how will you make it continuous? [7]

c. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, prove that

$$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$ [6]

d. Find all the points of local maxima and minima with their corresponding values of the function $f(x) = \sin x + \dfrac{1}{2}\cos 2x$, where $0 \leq x \leq \dfrac{\pi}{2}$. [6]