Spring Semester Examination 2017
Paro College of Education
Royal University of Bhutan
Paro

Module : MAT 205 (Linear Algebra) **Programme:** B.Ed(S) **Level** : II

Writing Time: Three Hours **Full Marks:** 100

Instructions : Do not write during the first 15 minutes. Use this time for reading the questions. You will get full three hours for answering the questions.Write the answers to all the questions in the answer sheets provided by the college.Read the directions to each section and to each question carefully before answering the questions. You are allowed to carry a scientific calculator ***fx-82 or fx-100*** beside other writing materials. You will be provided with graph sheets.

Instructions : This paper contains FIVE questions. You can answer any FOUR questions. All questions carry 25 marks each. Marks for each question or sub question are given in the brackets.

Question 1

a. Test coplanarity of the following vectors:

$$3\hat{i} + \hat{j} - \hat{k},\ 2\hat{i} - \hat{j} + 7\hat{k} \text{ and } 7\hat{i} - \hat{j} + 3\hat{k}$$ **(6)**

b. Draw the graph and shade the feasible region of the following system of linear inequalities : **(6)**

$$3x + 2y \leq 18,\ x + 2y \geq 3,\ 4y - 3x \leq 12,\ -x + 2y \geq -2,\ 5x + 4y = 20$$

c. Using properties of determinants, show that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$ **(6)**

d. Using the adjoint inverse method, solve the system of linear equations: **(7)**

$$x + y + z + w = 1,\ x - 2y + 2z + 2w = -6,\ 2x + y - 2z + 2w = -5,\ 3x - y + 3z - 3w = -3$$

Question 2

a. $ABCD$ is a parallelogram. If L and M are the mid-points of BC and DC respectively, then express $\vec{AL}$ and $\vec{AM}$ in terms of $\vec{AB}$ and $\vec{AD}$. Also, prove that $\vec{AL} + \vec{AM} = \frac{3}{2}\vec{AC}$. **(6)**

b. Solve $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$ **(6)**

c. Find the Eigen values and corresponding Eigen vectors for any one of the Eigen values for the matrix **(6)**

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

d. There is a factory located at each of the two places P and Q. From these locations, a certain commodity is delivered to each of the three depots situated at A, B and C. The weekly requirements of depots are 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are 8 and 6 units respectively. The cost of transportation per unit is given below :

From/To	**Cost (in Nu)**		
	A	B	C
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost? Solve this problem using Corner Point method of linear programming. **(7)**

Question 3

a. Using vector method, prove that a rhombus is a square, if the diagonals of a rhombus are equal in length. **(7)**

b. Solve the following systems of linear equations by Cramer's rule **(6)**

$$5x - 7y + z = 11,\ 6x - 8y - z = 15,\ 3x + 2y - 6z = 7$$

c. Using Row reduction method, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ **(6)**

d. A hospital dietician wishes to find the cheapest combination of two foods, A and B, that contains at least 0.5mg of thiamine and at least 600 calories. Each unit of food A contains 0.12mg thiamine and 100 calories, while each unit of food B contains 0.01mg of thiamine and 150 calories. If each food cost Nu.10 per units, how many units of each should be combined at a minimum cost? Solve this problem using Iso-cost method of linear programming. **(6)**

Question 4

a. Using vector method, prove that the area of a triangle is $\frac{1}{2}|\vec{a} \times \vec{b}|$, where $\vec{a}$ and $\vec{b}$ are the adjacent sides of the triangle. Using the proved formula, find the area of the triangle whose vertices are $A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -3, 1)$. **(7)**

b. By using determinant, solve the following system of equations: **(6)**

$$x + y - z = 0,\ x - 2y + z = 0,\ 3x + 6y - 5z = 0$$

c. Three shopkeepers A, B and C go to a store to buy stationery. A purchases 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs Nu.40, a pen costs Nu. 10 and pencil costs Nu.5. Use matrix to calculate each individual's amount paid to buy the stationeries. **(6)**

d. Solve the following linear programming problem graphically using Iso-profit methods. **(6)**
Maximize $Z = 5x + 7y$ subject to the constraints

$$x + y \leq 4,\ 3x + 8y \leq 24,\ 10x + 7y \leq 35,\ x \geq 0,\ y \geq 0$$

Question 5

a. Using vector, prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B$. **(7)**

b. Determine the consistence of the system of the equation using determinant and find solution if exist. **(6)**

$$3x - y + 2z = 3,\ 2x + y + 3z = 5,\ x - 2y - z = 1$$

c. Solve the following linear programming problem graphically, using Corner Point method : **(6)**
Minimize $Z = 20x + 10y$, subjected to the constraints

$$x + 2y \leq 40,\ 3x + y \geq 30,\ 4x + 3y \geq 60,\ x \geq 0,\ y \geq 0$$

d. Given the following communication matrix, answer the following questions.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

i. Draw a network diagram for the matrix. **(2)**

ii. Who can C talk to? **(1)**

iii. Who can B receive calls from? **(1)**

iv. Why is the main diagonal all zeroes? **(1)**

v. To whom D cannot call? **(1)**